Fall 2013

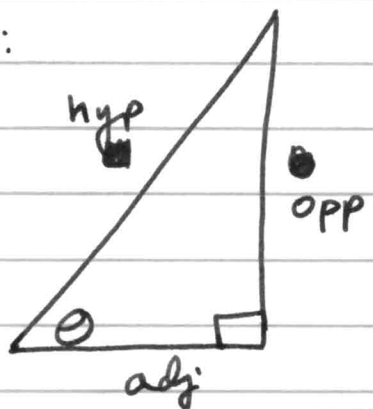
4.3 - Right-Angle Trig

Before most of our trig was done with triangles

Now most of our trig will be done with circles.

But first: a refresher!

Consider:



Define $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$ ← the vertical part

$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$ ← the horizontal part

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{\text{opp}}{\text{hyp}}}{\frac{\text{adj}}{\text{hyp}}} \cdot \frac{\text{hyp}}{\text{hyp}} = \frac{\text{opp}}{\text{adj}}$$

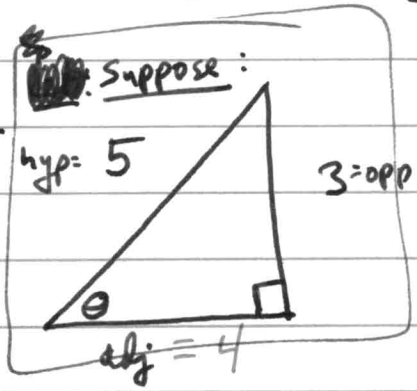
$\csc(\theta) = \frac{1}{\sin\theta} = \dots$

$\sec(\theta) = \frac{1}{\cos\theta}$

$\cot(\theta) = \frac{1}{\tan\theta}$

We can use Δ's to compute trig fns

Eg: Suppose:



Find all 6 trig values of θ

$(adj)^2 + (opp)^2 = (hyp)^2$

$adj^2 + 3^2 = 5^2$

$adj^2 + 9 = 25$

$(adj)^2 = 16$

$adj = 4$ ← because side length is positive

So: $\sin(\theta) = \frac{opp}{hyp} = \frac{3}{5}$

$\cos(\theta) = \frac{adj}{hyp} = \frac{4}{5}$

$\tan(\theta) = \frac{opp}{adj} = \frac{3}{4}$

$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{5}{3}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{5}{4}$

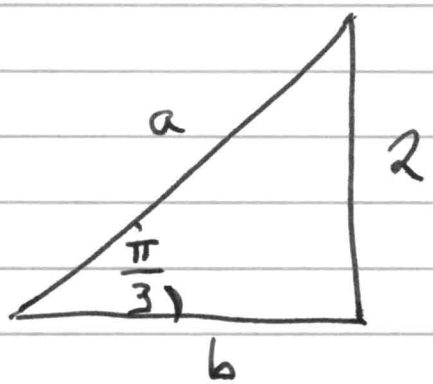
$\cot(\theta) = \frac{1}{\tan\theta} = \frac{4}{3}$

Some Common Angles:

Degrees	0	30°	45°	60°	90°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Find the value of a and b

Eg:



$\sin\left(\frac{\pi}{3}\right) = \frac{opp}{hyp} = \frac{2}{a}$

Also $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

so $\frac{2}{a} = \frac{\sqrt{3}}{2}$

SOLVE for a: $a\sqrt{3} = 4$

$a = \frac{4}{\sqrt{3}}$

$\tan\left(\frac{\pi}{3}\right) = \frac{opp}{adj} = \frac{2}{b}$

also

$\tan\left(\frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \sqrt{3}$

so

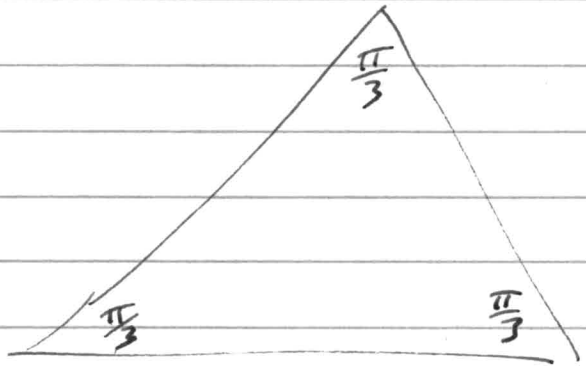
$\frac{2}{b} = \sqrt{3}$

So $b = \frac{2}{\sqrt{3}}$

Triangle Review

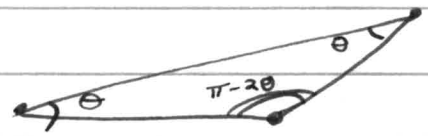
- angles in $\triangle$ add to π ($\frac{1}{2}$ of a circle)
- 2 impt. types of $\triangle$'s

① all angles equal $\Rightarrow$ all $\triangle$'s are $\frac{\pi}{3}$

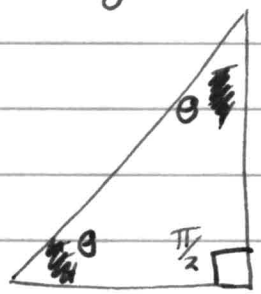


5-10

eg:



② two angles equal. If 3rd $\triangle$ is $\frac{\pi}{2}$, Then



$2 \cdot (\theta) + \frac{\pi}{2} = \pi$

$2 \cdot (\theta) = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

first

labs

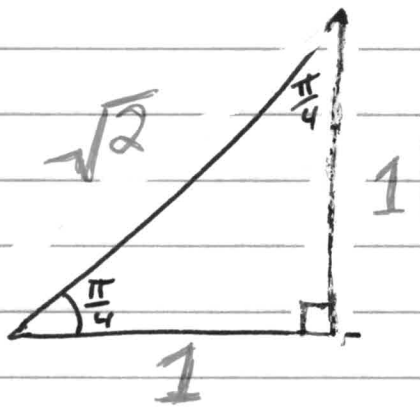
compute $\sin\left(\frac{\pi}{4}\right)$ & $\cos\left(\frac{\pi}{4}\right)$

Need: a right triangle (one $\measuredangle = \frac{\pi}{2}$) with one angle $= \frac{\pi}{4}$.

Remember: the sum of $\measuredangle$'s in a triangle is π ($\frac{1}{2}$ of a circle).

so missing $\measuredangle + \frac{\pi}{4} + \frac{\pi}{2} = \pi$

so missing $\measuredangle = \frac{\pi}{4}$



We can now read trig fns off the $\triangle$:

$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$

$\cos\left(\frac{\pi}{4}\right) = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}$

$\tan\left(\frac{\pi}{4}\right) = \frac{opp}{adj} = 1$

Now need the sides

to make things nice, lets set the two equal length sides $= 1$

Notice $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

then $1^2 + 1^2 = hyp^2$

$2 = hyp^2$

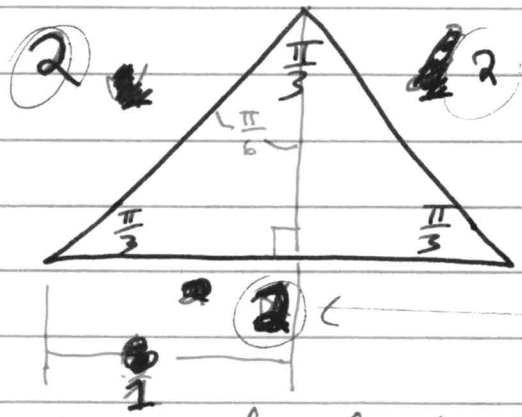
so $hyp = \sqrt{2}$

Now lets compute $\sin\left(\frac{\pi}{3}\right)$ & $\cos\left(\frac{\pi}{3}\right)$

Need a $\triangle$ with a right $\angle$ and a $\frac{\pi}{3}$ angle.

But first, lets

to draw a triangle with 3 equal angles $= \frac{\pi}{3}$



let all the equal sides have length 2
(this will make the things nicer)

Need: a $\triangle$ with a right $\angle$ and a $\frac{\pi}{3}$ angle.

NOTICE: we can cut the above $\triangle$ in half!

$adj^2 + opp^2 = hyp^2$

$1 + opp^2 = 2^2$

$opp^2 = 4 - 1 = 3$

$opp = \sqrt{3}$

Read off trig fns: $\sin\left(\frac{\pi}{3}\right) = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{adj}{hyp} = \frac{1}{2}$

To find $\sin\left(\frac{\pi}{6}\right)$ & $\cos\left(\frac{\pi}{6}\right)$

need a right triangle

with a $\frac{\pi}{6}$ angle

Notice $\frac{\pi}{6} + \text{missing } \measuredangle + \frac{\pi}{2} = \pi$

$\text{missing } \measuredangle = \pi - \frac{3}{3}\frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{6\pi - 3\pi - \pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

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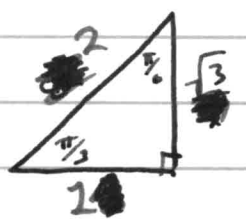
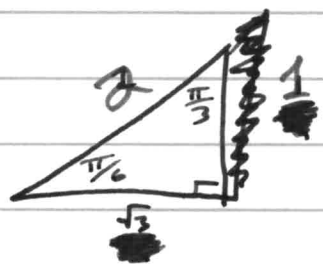
But we already have this triangle!

flip



to get



so $\sin\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$

$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$

You see

Your friend, who is 6' tall,

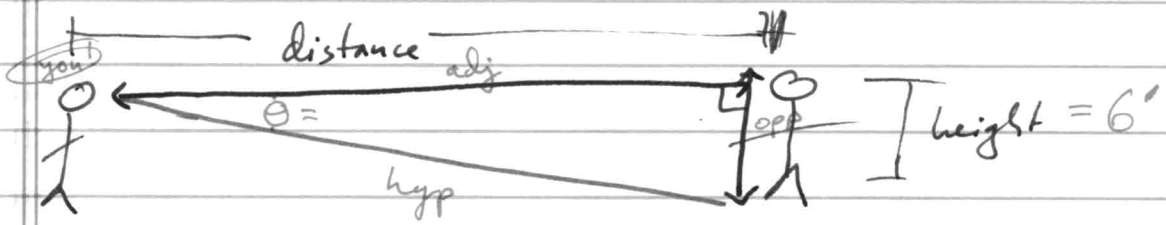
across the park.

You want to know how far away they are

but all you have is a protractor

what do you do?

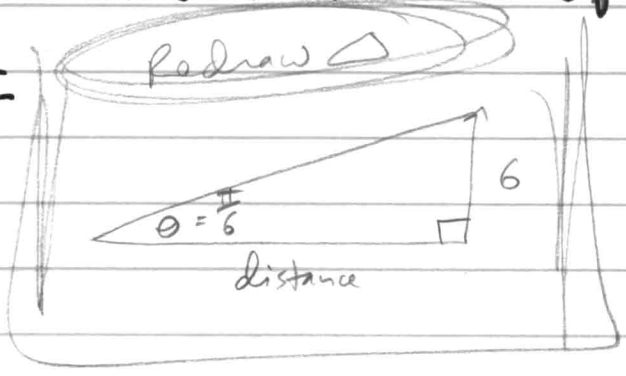
Ai Draw a picture (& a triangle)



Idea: measure the ∠ you see between their head & feet

you measure $\theta = \frac{\pi}{6}$

redraw △



$\tan\theta = \frac{opp}{adj}$

$\tan\left(\frac{\pi}{6}\right) = \frac{6'}{adj}$

$$adj = \frac{6'}{\tan\left(\frac{\pi}{6}\right)} = \frac{(6)\left(\frac{\sqrt{3}}{\sqrt{3}}\right)}{\frac{1}{\sqrt{3}}} = 6\cdot\sqrt{3} \approx 10'$$

your friend is about 10' away. they must think you are crazy.